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## On axion-mediated macroscopic forces again

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### Abstract

In a supersymmetric unified theory or in a generic model where a large neutron electric dipole moment  $d_N$  is expected, close to the present bound, we estimate the relation  $g_{aNN} = 10^{-21 \pm 1} (d_N / 10^{-25} e \cdot \text{cm}) (10^{10} \text{ GeV} / f_a)$  between  $d_N$  itself, the scalar coupling  $g_{aNN}$  to nucleons of the axion, assumed to exist, and the breaking scale,  $f_a$ , of the Peccei-Quinn symmetry. Newly developing techniques to search for sub-cm macroscopic forces might reveal a signal due to axion exchange at least in a favorable range of  $f_a$ .

**1** Electric Dipole Moments (EDMs) of the electron and the neutron at the border of the present limits are expected in supersymmetric unified theories with supersymmetry breaking transmitted by supergravity couplings [1]. Such EDMs are generated from CKM-like phases via one loop diagrams involving sfermion and gaugino-higgsino exchanges at the weak scale. In these models, however, as in most other cases where a sizeable one-loop quark-EDM occurs [2], similar diagrams give also rise to a strong CP violating angle,  $\theta_{\text{QCD}}$ , which is *too large* if not counteracted by an appropriately tuned initial condition. Therefore, especially in these models, a Peccei-Quinn [3] (PQ) solution of the strong CP problem is called for, leading to a so-called “invisible” axion [4].

Unfortunately, such a solution of the strong CP problem is as elegant as it is experimentally elusive. Nevertheless, rightly so, a number of serious experimental proposal for axion detection have been made. Among them, the search for a CP-violating macroscopic force mediated by axion exchange [5] is the possibility that we want to reconsider in this letter.

**2** Crucial parameters to this effect are the mass  $m_a$  and the *scalar* coupling  $g_{aNN}$  of the axion to the nucleons. Both  $m_a$  and  $g_{aNN}$  are inversely proportional to the PQ symmetry breaking scale, conventionally called  $f_a$ , which is constrained to lie in the range  $10^7 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$  [6]. In terms of the quark masses,  $m_u$ ,  $m_d$ , and of the pion mass and decay constant,  $m_\pi$  and  $f_\pi$ , it is

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} = \frac{1}{0.02 \text{ cm}} \frac{10^{10} \text{ GeV}}{f_a}. \quad (1)$$

Furthermore, the required weak CP violation leads to a residual dynamically determined  $\theta_{\text{QCD}} \neq 0$ , which, in turn, induces an axion-nucleon scalar coupling [5] (disregarding the relatively small but phenomenologically potentially important difference between the axion-proton and the axion-neutron couplings)

$$g_{aNN}[\theta_{\text{QCD}}] = \frac{\theta_{\text{QCD}}}{f_a} \frac{m_u m_d}{m_u + m_d} \langle N | \bar{u}u + \bar{d}d | N \rangle \approx 3 \cdot 10^{-12} \theta_{\text{QCD}} \frac{10^{10} \text{ GeV}}{f_a} \quad (2)$$

In view of the limit set by the null results of the measurements of the neutron EDM so far [7],  $\theta_{\text{QCD}} \lesssim 10^{-9}$  [8], the Yukawa-type interaction induced by one-axion exchange is therefore bound to be small, at about the level of gravity or lower. Maybe not so small, however, to escape detection in experiments proposed [9] or conceivable [5] to search for new sub-cm forces. The potentiality of axion searches by looking for axion-mediated macroscopic forces has been already emphasized in ref. [10].

All this makes it interesting to ask, in supersymmetric unified theories, at what level  $g_{aNN}[\theta_{\text{QCD}}]$  actually sets in (what is  $\theta_{\text{QCD}}?$ ), or, more importantly, what is the value of  $g_{aNN}$  at all, including any possible effect from other CP violating operators. To our knowledge these questions have been addressed and satisfactorily answered [11] only in the case of the Standard Model, reaching a pretty negative conclusion: in the SM  $g_{aNN}$  is too small to be of any interest. One should not forget, however, that CP violation in the electroweak sector of the SM is screened enough that, even in absence of an axion, the radiative contributions to the  $\theta_{\text{QCD}}$  parameter are also negligibly small [12].

**3** Of relevance to the question under consideration is the effective lagrangian just above the chiral symmetry breaking scale,  $\Lambda_\chi$ , including the axion interactions and the flavour-conserving CP-violating operators. As we shall see, it is useful to consider at the same time the axion coupling and the neutron EDM, since the relation between the two quantities is largely model independent.

Following refs [11, 13], we consider a non linear realization of the PQ symmetry where the axion field  $a$  transforms as

$$a \rightarrow a + \text{cte},$$

whereas all the matter fields remain invariant. In this basis the axion would have no non-derivative coupling at all, if it were not for the anomalous term

$$- \frac{a}{f_a} \frac{\alpha_s}{8\pi} G^{\mu\nu} \tilde{G}_{\mu\nu}. \quad (3)$$

This term too can be eliminated by a chiral rotation acting on the quark fields  $q = (u, d)^T$

$$q \rightarrow \exp(-iQ_A \gamma_5 \frac{a}{f_a}) q \quad (4)$$

at the price of introducing axion dependence in the chirality breaking quark operators. In terms of the quark mass matrix  $m_q$ , the matrix  $Q_A$  is

$$Q_A = \frac{1}{2} \frac{m_q^{-1}}{\text{Tr } m_q^{-1}}, \quad (5)$$

chosen to eliminate mass mixing between the axion and the pseudoscalar mesons. In the effective lagrangian it is therefore useful to distinguish, among the CP violating operators, those ones that respect chiral symmetry, generically denoted by  $\mathcal{O}^\chi$ , from those that break chiral symmetry,  $\mathcal{O}^{\text{nc}}$ . After elimination of the anomaly term (3) by the chiral rotation (4), the relevant axion dependence resides in the mass term

$$\overline{q_L} e^{-iQ_A a/f_a} M e^{-iQ_A a/f_a} q_R \quad (6)$$

and in  $\mathcal{O}^{\text{nc}}$  only.

Examples of  $\mathcal{O}^\chi$  are the Weinberg 3-gluon operator [14]

$$G_\nu^\mu G_\rho^\nu \tilde{G}_\mu^\rho, \quad (7a)$$

six-quark operators like

$$[\overline{u_L} \gamma^\mu d_L \cdot \overline{d_L} \gamma_\mu] \not{D} [\gamma^\nu s_L \cdot \overline{s_L} \gamma_\nu u_L], \quad (7b)$$

or the interplay between two flavour violating operators

$$(\overline{s_L} \gamma_\mu d_L)(\overline{u} \gamma_\mu u) \quad \text{and} \quad (\overline{s_L} \gamma_\mu d_L)(\overline{d} \gamma_\mu d). \quad (7c)$$

The prototype example of  $\mathcal{O}^{\text{nc}}$  is, on the other hand, the ChromoElectric Dipole Moment (CEDM) operator

$$d_q^{\text{QCD}} \times \frac{1}{2} (\bar{q} \sigma_{\mu\nu} \gamma_5 q) G^{\mu\nu}. \quad (8)$$

To obtain the axion-nucleon scalar coupling and the neutron EDM one has to cross the chiral symmetry breaking scale  $\Lambda_\chi \approx 1 \text{ GeV}$  and go to the confinement scale, just above  $\Lambda_{\text{QCD}}$ . This we do, as in ref. [15], by use of Naive Dimensional Analysis (NDA). This technique is appropriate to the general discussion that we want to make and is not too inaccurate, given our presently limited understanding of low energy QCD. The crucial notion of NDA is that the reduced coupling  $\bar{g}$  appearing in front of an operator  $\mathcal{O}$ , that one seeks to calculate in the effective hadronic theory, is given by the product of the reduced couplings of the operators that produce  $\mathcal{O}$  in the effective lagrangian involving quarks and gluons. For an operator with dimensionful coupling  $g$ , of dimension  $d$  in mass and involving  $n$  fields, the dimensionless reduced coupling  $\bar{g}$  is

$$\bar{g} \equiv g \cdot (4\pi)^{2-n} \Lambda_\chi^{d-4}. \quad (9)$$

**4** As mentioned, we consider at the same time the axion-nucleon scalar coupling,  $g_{aNN}$ , and the neutron EDM,  $d_N$ . Notice that they not only both violate CP but also have the same chiral properties.

As source of CP violation, let us take first the quark CEDM  $d_q^{\text{QCD}}$ . The CEDM operator carries axion dependence, since it breaks chiral symmetry; it has in fact the same chiral properties of  $g_{aNN}$  and  $d_N$  themselves.

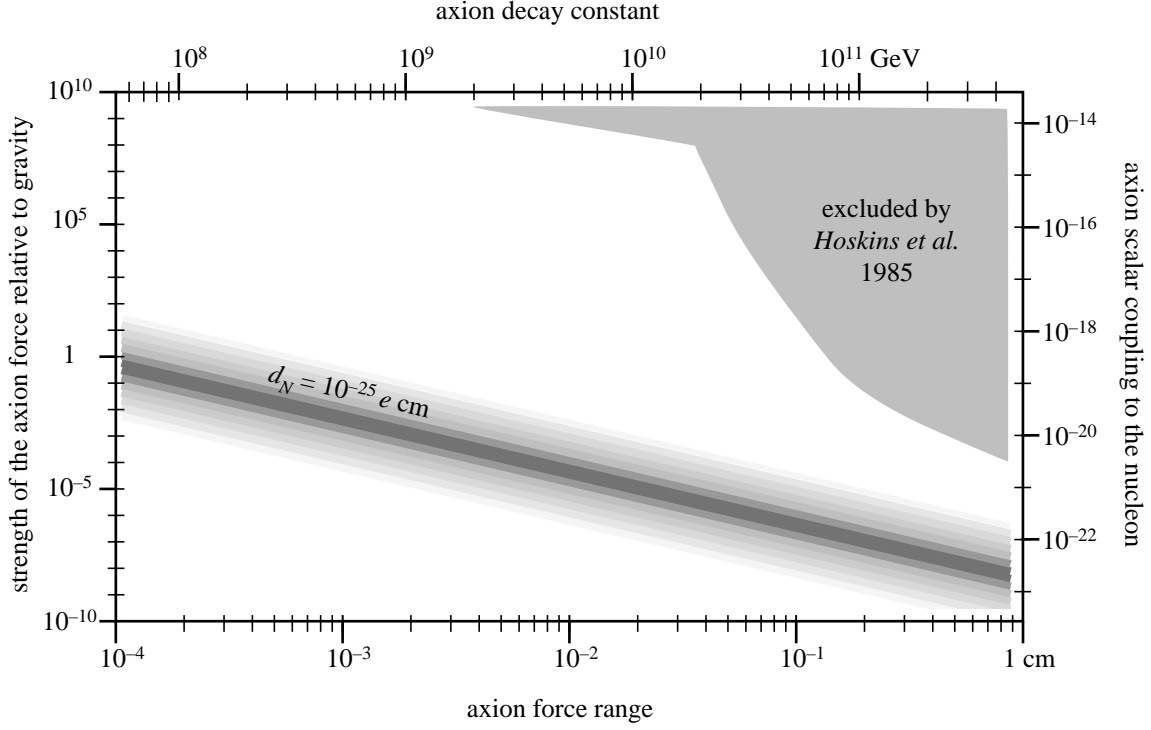


Figure 1: *estimates of the axion force strength relative to gravity for  $d_N = 10^{-25} e \cdot \text{cm}$ . Also shown is the presently excluded region.*

By means of NDA, it is immediate to get the contributions to  $d_N$  and  $g_{aNN}$  induced by  $d_q^{\text{QCD}}$ . It is

$$d_N[d_q^{\text{QCD}}] \approx e \frac{g_s}{(4\pi)^2} \langle d_q^{\text{QCD}} \rangle, \quad (10a)$$

$$g_{aNN}[d_q^{\text{QCD}}] \approx \frac{\Lambda_\chi^2}{f_a} \frac{g_s}{(4\pi)^2} \langle 2Q_A d_q^{\text{QCD}} \rangle, \quad (10b)$$

where  $\langle \dots \rangle$  denotes a weighted sum, with coefficients of order unity, over the up and down quarks. From (10a), since  $\langle d_q^{\text{QCD}} \rangle \approx \langle 2Q_A d_q^{\text{QCD}} \rangle$ , we have

$$g_{aNN}[d_q^{\text{QCD}}] \approx \frac{d_N[d_q^{\text{QCD}}]}{e} \frac{\Lambda_\chi^2}{f_a}. \quad (11)$$

It should be clear, however, that a relation like (11) holds, within the limits of NDA, for any operator, or combination of operators, of the type  $\mathcal{O}^{\text{nx}}$ , involving quarks and gluons only,

$$g_{aNN}[\mathcal{O}^{\text{nx}}] \approx \frac{d_N[\mathcal{O}^{\text{nx}}]}{e} \frac{\Lambda_\chi^2}{f_a}. \quad (12)$$

A different relation holds between  $g_{aNN}$  and  $d_N$  generated by the standard quark EDMs  $d_q$ , since

$$d_N[d_q] \approx \langle d_q \rangle \quad (13a)$$

$$g_{aNN}[d_q] \approx \frac{e}{(4\pi)^2} \frac{\Lambda_\chi^2}{f_a} \langle 2Q_A d_q \rangle. \quad (13b)$$

In the models of interest, however,  $d_N[d_q^{\text{QCD}}] \gtrsim d_N[d_q]$ , so that  $g_{aNN}[d_q] < g_{aNN}[d_q^{\text{QCD}}]$ . Eq. (12) remains therefore appropriate even with the inclusion in  $\mathcal{O}^{\text{nx}}$  of the quark EDMs.

Let us now consider  $g_{aNN}$  and  $d_N$  generated by CP-violating chirally invariant operators  $\mathcal{O}^\chi$ . In this case an asymmetry occurs between  $g_{aNN}$  and  $d_N$ . Although, to generate both  $g_{aNN}$  and  $d_N$ ,  $\mathcal{O}^\chi$  must be supplemented by a chirality breaking operator, the most economic way for  $d_N$  is through the so called “soft” quark mass

$$m_{\text{soft}} \approx g_s^2 \frac{\langle \bar{q}q \rangle}{p^2} \approx 350 \text{ MeV} \quad (14)$$

(becoming actually soft only at momenta  $p \gg \Lambda_\chi$ ), whereas  $g_{aNN}$  comes through the “current” mass term (6) in order to introduce also the required  $a$ -dependence. Therefore eq. (12) is corrected by a relative factor  $\langle 2Q_A M \rangle / m_{\text{soft}}$  or

$$g_{aNN}[\mathcal{O}^\chi] \approx \frac{m_u m_d}{m_{\text{soft}}(m_u + m_d)} \frac{d_N[\mathcal{O}^\chi]}{e} \frac{\Lambda_\chi^2}{f_a}. \quad (15)$$

Eq.s (12) and (15) represent our estimates for the relation between  $g_{aNN}$  and  $d_N$  in a generic model, which can of course be summarized as

$$g_{aNN} \approx \frac{\Lambda_\chi^2}{e f_a} \left\{ d_N[\mathcal{O}^{\text{nx}}] + \frac{m_u m_d}{m_{\text{soft}}(m_u + m_d)} d_N[\mathcal{O}^\chi] \right\}. \quad (16)$$

The SM is a prototype of models where  $d_N(\mathcal{O}^\chi)$  dominates  $d_N$  (which is, mostly for the same reason, rather small) [12, 16]. Consequently

$$g_{aNN}^{\text{SM}} \approx \frac{\Lambda_\chi^2}{e f_a} \frac{m_u m_d}{m_{\text{soft}}(m_u + m_d)} d_N^{\text{SM}} \approx 10^{-30 \pm 1} \frac{d_N^{\text{SM}}}{10^{-32} e \cdot \text{cm}} \frac{10^{10} \text{ GeV}}{f_a},$$

too small to be of any experimental interest [11]. On the other hand, for the Unified Supersymmetric Models (USMs) or for a generic model where  $d_N$  is dominated by  $d_N[\mathcal{O}^{\text{nx}}]$  (and possibly large, because of this very reason)

$$g_{aNN}^{\text{USM}} \approx 10^{-21 \pm 1} \frac{d_N^{\text{USM}}}{10^{-25} e \cdot \text{cm}} \frac{10^{10} \text{ GeV}}{f_a}. \quad (17)$$

We have explicitly indicated the uncertainty that must be attributed to our estimates, essentially due to the limited control of QCD in the infrared regime.

Taking into account of the expectations for  $d_N^{\text{USM}}$ , which saturate the present bound [1], *the value of  $g_{aNN}$  in eq. (17) leads to a signal at the border of the sensitivity of planned or conceived experiments to search for macroscopic sub-cm forces*, at least in a favorable range of  $f_a$  [9, 5].

For the dimensionless ratio between the strength of the axion induced gravity-like force and gravity itself, one has

$$\frac{\mathcal{F}_{\text{axion}}}{\mathcal{F}_{\text{gravity}}} = + \frac{g_{aNN}^2 / 4\pi}{G_N m_N^2} = 10^{-5 \pm 2} \left( \frac{d_N}{10^{-25} e \cdot \text{cm}} \right)^2 \left( \frac{10^{10} \text{ GeV}}{f_a} \right)^2, \quad (18)$$

as represented in fig. 1. Monopole-dipole effects might also be relevant [5]. Eötvös-type experiments, if possible in the sub-cm range, would of course also be of great significance. The importance of looking for such effects cannot be possibly overestimated. It is interesting to notice that the relevance of similar types of experiments has also been recently emphasized in connection with the moduli fields characteristic of superstring theories [17].

To conclude, we notice that the contribution to  $g_{aNN}$  from the  $\theta_{\text{QCD}}$ -parameter, which started our discussion, does not alter in any significant way the result in eq. (16). Actually, the very distinction between  $g_{aNN}[\theta_{\text{QCD}}]$  and the other contributions to  $g_{aNN}$  is not even unambiguously defined in the hadronic lagrangian because  $\theta_{\text{QCD}}$  itself is not, unlike the case for the basic QCD lagrangian, since several terms will generally have independent phases, each of the same order.

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